THE ROLE OF PARTICLE COLLISIONS

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The effect of collisions between monodisperse particles on their entrainment by a flow is investigated by treating the system of suspended particles as a continuum.

The existing analytic methods of investigating the motion of particles suspended in carrier media are based on the individual trajectories of the noninteracting particles. However, in a number of cases this approach is not satisfactory.

The problem of particle acceleration by a flow in a straight channel was investigated in [1], where the following particle equation of motion was constructed:

$$m\frac{du}{dt}=\zeta\frac{\pi d^2}{4}\frac{\rho(v-u)^2}{2}\pm mg. \qquad (1)$$

Integration of this equation gives a distribution of velocity u and particle concentration $\varkappa$ along the length of the channel x that differs considerably from the experimental distribution. In [1] this discrepancy is attributed to the effect of particle collisions with each other and with the channel walls. It is worthwhile obtaining an anlytic estimate of the part played by these collisions.

Obviously, the effect of collisions increases with the particle concentration $\varkappa$. Accordingly, we consider values $\varkappa > \varkappa_0$ at which it is reasonable to treat the particle system as a continuum-gas analogy. In this case the lower limit $\varkappa_0$ is determined by the condition $\lambda \ll L$, where L is the characteristic dimension of the flow region (in our case the channel width), and λ is the mean distance between particles (of the order of $(m/\varkappa_0)^{1/3}$.

§1. For a system of identical particles, we write the equations of motion as for a simple gas with a distributed body force f due to interaction between the particles and the carrier medium. The dynamic equation is

$$\varkappa\frac{d}{dt}\langle \mathbf{u}\rangle=-\operatorname{grad}p+\langle \mathbf{f}\rangle+\varkappa\mathbf{g}, \qquad (2)$$

the continuity equation is

$$\frac{d\varkappa}{dt}+\varkappa\operatorname{div}\langle \mathbf{u}\rangle=0, \qquad (3)$$

and the energy equation is

$$\varkappa\frac{d}{dt}\left(c_vT+\frac{\langle \mathbf{u}\rangle^2}{2}\right)=\langle \mathbf{f}\cdot\mathbf{u}\rangle-\operatorname{div}(p\langle \mathbf{u}\rangle) \qquad (4)$$

(they can easily be obtained on the basis of general principles [2]).

We determine T with the equation from the kinetic theory of gases, which presupposes that the energy of random motion of the particles is equally distributed over the degrees of freedom [3]

$$m(\langle u^2\rangle-\langle u\rangle^2)=3kT.$$

We have the analog of the specific heat $c_V = (3/2)(k/m)$ and of the equation of state $p = \varkappa(k/m)T$.

We determine the force **f** in the Stokes approximation. The force acting on an individual particle is $\mathbf{f}_1 = 3\pi d\mu(\mathbf{v}-\mathbf{u})$. There are $n = \varkappa/m$ particles per unit volume; consequently, the mean body force is

$$\langle \mathbf{f}\rangle=3\pi d\mu\frac{\varkappa}{m}(\mathbf{v}-\langle \mathbf{u}\rangle). \qquad (5)$$

Correspondingly, in the energy equation

$$\langle \mathbf{f}\cdot\mathbf{u}\rangle=3\pi d\mu\frac{\varkappa}{m}\langle(\mathbf{v}-\mathbf{u})\cdot\mathbf{u}\rangle=$$
$$=3\pi d\mu\frac{\varkappa}{m}(\mathbf{v}\cdot\langle \mathbf{u}\rangle-\langle \mathbf{u}\rangle^2-2c_vT). \qquad (6)$$

Using (5) and (6) and discarding the averaging sign, we write the equations as

$$\varkappa\frac{d\mathbf{u}}{dt}=-\operatorname{grad}p+\alpha\varkappa(\mathbf{v}-\mathbf{u})+\mathbf{g}\varkappa,\quad \alpha=\frac{3\pi d\mu}{m},$$
$$\frac{d\varkappa}{dt}+\varkappa\operatorname{div}\mathbf{u}=0,$$
$$\varkappa\frac{d}{dt}\left(c_vT+\frac{u^2}{2}\right)=\varkappa\alpha(vu-u^2-2c_vT)-\operatorname{div}(pu),$$
$$p=\varkappa\frac{k}{m}T. \qquad (7)$$

Equations (7) correspond to the equations of motion of a gas in the absence of dissipative effects (i.e., the analogs of viscosity and thermal conductivity are equal to zero). When $\alpha = 0$ Eqs. (7) become the equations of motion of a gas; accordingly, at small α mutual simulation of the gas and particle flows is possible. At large α additional effects differentiating the behavior of particles and gases appear.

§2. We consider the stationary one-dimensional motion of particles in a straight channel, $(d/dt) = u(d/dx)$, whose equations are

$$\varkappa u\frac{du}{dx}=-\frac{dp}{dx}+\alpha\varkappa(v-u),$$
$$\varkappa u=\text{const},\quad p=\varkappa\frac{k}{m}T,$$
$$\varkappa u\frac{d}{dx}\left(c_vT+\frac{u^2}{2}\right)=$$
$$=\varkappa\alpha(vu-u^2-2c_vT)-\frac{d}{dx}(pu),$$
$$c_v=\frac{3}{2}\frac{k}{m}. \qquad (8)$$

The third equation of system (8) can be transformed using the first two equations. In fact, we have

$$\varkappa u \frac{d}{dx}(c_v T) + u\left[\varkappa u \frac{du}{dx} - \varkappa\alpha(v-u)\right] + $$
$$+ 2\alpha\varkappa c_v T + \frac{d}{dx}(pu) = 0$$

or

$$u\frac{dT}{dx} + \frac{2}{3}T\frac{du}{dx} + 2\alpha T = 0. \tag{9}$$

Equation (9) can be integrated in the following form:

$$T = T_0\left(\frac{u_0}{u}\right)^{2/3}\exp\left(-2\alpha\int_0^x \frac{dx}{u}\right). \tag{10}$$

It follows from (10) that as the particles are accelerated, $u \to v$, T decreases, i.e., $dT/dx < 0$, and, in the gas analogy, the interaction of the particles with the flow is equivalent to "cooling" of the particle system. In the particular case when $\alpha = 0$ we obtain

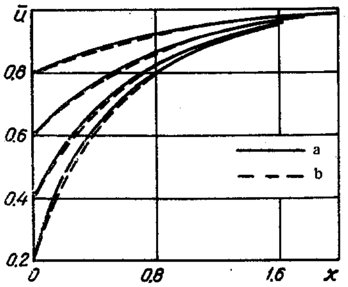

Fig. 1. Effect of $\bar{u}_0$ on particle acceleration at $\bar{T}_0 = 0$: a) from the equation $\bar{u} = 1 - (1 - \bar{u}_0) \times \times \exp(-\gamma\bar{x})$; b) exact solution.

$$T = T_0\left(\frac{u_0}{u}\right)^{2/3} = T_0\left(\frac{x}{x_0}\right)^{2/3},$$

which corresponds to the adiabatic law with $c_p/c_V = 5/3$ (monatomic gas).

In the general case system (8) is not integrable. We find an approximate solution in the form

$$u = v - (v - u_0)\exp(-\gamma x),$$
$$T = T_0\exp(-\beta x). \tag{11}$$

First, for convenience, we write the equations in dimensionless form

$$\frac{d\bar{u}}{d\bar{x}} = -\frac{d}{d\bar{x}}\left(\frac{\bar{T}}{\bar{u}}\right) + \left(\frac{1}{\bar{u}} - 1\right),$$
$$\bar{u}\frac{d\bar{T}}{d\bar{x}} + \frac{2}{3}\bar{T}\frac{d\bar{u}}{d\bar{x}} + 2\bar{T} = 0, \tag{12}$$

where $\bar{u} = u/v$, $\bar{T} = (k/m)(T/v^2)$, $\bar{x} = \alpha x/v$.

To determine γ and β we integrate Eqs. (12) with respect to x from 0 to infinity and substitute Eqs. (11). Considering that $u \to 1$, $\bar{T} \to 0$ as $\bar{x} \to \infty$, we obtain

$$1 - \bar{u}_0 = \frac{\bar{T}_0}{\bar{u}_0} + \frac{1}{\gamma}\ln\frac{1}{\bar{u}_0},$$

$$-1 + (1 - \bar{u}_0)\frac{\beta + \frac{2}{3}\gamma}{\beta + \gamma} + \frac{2}{\beta} = 0.$$

Now, we determine

$$\gamma = \frac{\ln\frac{1}{\bar{u}_0}}{1 - \bar{u}_0 - \frac{\bar{T}_0}{\bar{u}_0}},$$

$$\beta = \frac{-\left(\frac{2}{3}\bar{u}_0\gamma + \frac{1}{3}\gamma - 2\right)}{2\bar{u}_0} - \frac{\sqrt{\left(\frac{2}{3}\bar{u}_0\gamma + \frac{1}{3}\gamma - 2\right)^2 + 4\gamma\bar{u}_0}}{2\bar{u}_0}. \tag{13}$$

It is easy to verify that at these values of γ and β Eqs. (11) give a fairly good approximation of the actual behavior of u and T, at least for small T_0.

For comparison, Fig. 1 presents values of $\bar{u}$ as a function of $\bar{x}$ at $\bar{T}_0 = 0$ calculated from (11) and from the exact solution of the equation $\bar{u}(d\bar{u}/dx) = 1 - \bar{u}$ corresponding to $\bar{T}_0 \equiv 0$.

The effect of $\bar{T}_0$ on $\bar{u}(\bar{x})$ is illustrated in Fig. 2. Clearly, the greater $\bar{T}_0$, the more intense the acceleration of the particles by the flow.

As $\bar{T}_0$ increases, the accuracy of Eqs. (11) decreases, because the variation of $\bar{u}$ and $\bar{T}$ deviates from the exponential. To estimate the effect of large values of $\bar{T}_0$ we rewrite the first of Eqs. (12), substituting $d\bar{T}/d\bar{x}$ from the second equation.

$$\frac{d\bar{u}}{d\bar{x}} = \frac{\frac{1}{\bar{u}} - 1 + \frac{2\bar{T}}{\bar{u}^2}}{1 - \frac{5}{3}\frac{\bar{T}}{\bar{u}^2}}. \tag{14}$$

From (14) it follows that when $\bar{u}_0 < 1$, $\bar{T}_0 < (3/5)\bar{u}_0^2$, $\left.\frac{d\bar{u}}{d\bar{x}}\right|_{x=0} > 0$ and the particles are accelerated by the flow (Fig. 2). When $\bar{T}_0 > (3/5)\bar{u}_0^2$, $\left.\frac{d\bar{u}}{dx}\right|_{x=0} < 0$, i.e., the particles are decelerated and hence the exponential

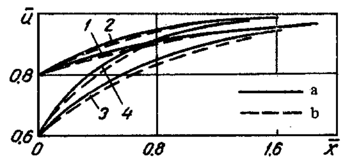

Fig. 2. Effect of $\bar{u}_0$ and $\bar{T}_0$ on the particle acceleration: 1) $\bar{u}_0 = 0.8$, $\bar{T}_0 = 0$; 2) 0.8 and 0.01; 3) 0.6 and 0; 4) 0.6 and 0.01; a) from the equation $\bar{u} = 1 - (1 - \bar{u}_0)\exp(-\gamma\bar{x})$, where γ is obtained from (13); b) approximate solution of linearized equations (8) at small $\bar{u}_0$ and T_0: $\bar{u} = 1 - (1 - \bar{u}_0)\exp(-\bar{x}) + 2\bar{T}_0[\exp(-\bar{x}) - \exp(-2\bar{x})]$.

approximation (11) does not hold. If the random and mean motions have comparable velocities, the nature of the particle acceleration will depend strongly on $\bar{T}_0$. Obviously, this will be particularly important at small $\bar{u}_0$, when slight changes in $\bar{T}_0$ may seriously modify the form of the relation $\bar{u} = \bar{u}(\bar{x})$ and hence $\varkappa(\bar{x}) = \mathrm{const}/\bar{u}(\bar{x})$.

Thus, one of the criteria determining the particle acceleration process must be the quantity

$$M = \frac{5}{3}\frac{\bar{T}_0}{\bar{u}_0^2} = \frac{5}{3}\frac{k}{m}\frac{T_0}{u_0^2} = \frac{c_p}{c_v}\frac{k}{m}\frac{T_0}{u_0^2},$$

which in a certain sense is analogous to the square of the Mach number (we recall that in gasdynamics the speed of sound is $a = ((c_p/c_v)(k/m)T)^{1/2}$).

When M = 1 the quantity $d\bar{u}/d\bar{x} = \infty$, i.e., there is a velocity jump and, as distinct from the case of gases, "sonic" motion along a straight channel is impossible.

In conclusion, we note that the M number may not be the only criterion. Taking into account the analog of dissipative processes (viscosity, thermal conductivity) must lead to additional criteria associated with the transverse dimensions of the channel. The particle size distribution should also be an important criterion, since different degrees of entrainment of the particles by the flow should lead to additional collisions. A detailed examination of these criteria is possible only on the basis of the methods of statistical mechanics.

NOTATION

m is the mass of a particle; d is the particle diameter; ξ is the particle aerodynamic drag coefficient; ρ is the density of carrier fluid; g is the acceleration of gravity; $\varkappa$ is the particle density (concentration); x is the coordinate along channel; u is the particle velocity or the mean velocity of particle flow; v is the carrier flow velocity; p, T, c_v are the analogs of pressure, temperature, and specific heat characterizing the intensity of random motion of the particles; k is the Boltzmann's constant; f is the Stokes force; μ is the dynamic viscosity of the fluid; $\alpha = 3\pi d\mu/m$; u_0 and T_0 are the mean velocity and "temperature" of particle system at x = 0, respectively; and γ, β are parameters. The subscript 0 denotes x = 0; barred quantities are dimensionless; the symbol $\langle\rangle$ denotes averaging of the particle parameters over unit volume.

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